Math 1B Quiz 1 Version 4

Wed Apr 18, 2018

NAME YOU

SCORE: ______ / 30 POINTS

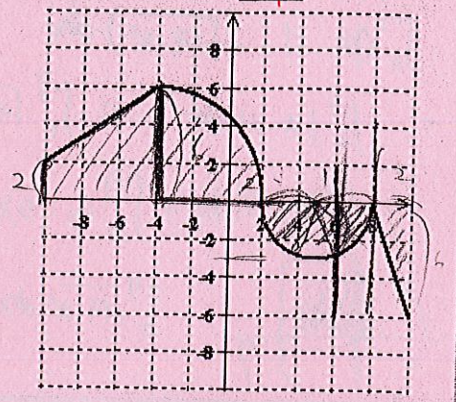
1. **No calculators allowed**
2. **Simplify all answers unless stated otherwise**
3. **Show proper calculus level work to justify your answers**

The graph of function f is shown on the right.

The graph consists of a diagonal line, arcs of 2 circles, then another diagonal line.

? ?

SCORE: 4 / 4 PTS



[a] Evaluate $\int_{-10}^{10} f(x)\,dx$.

NOTE: You must show the arithmetic expression that you used to get your answer.

$$\int_{-10}^{10} f(x)\,dx = \frac{(2+6)6}{2} + \frac{1}{4}\cdot 36\pi - \frac{1}{2}9\pi - 2\cdot 6\cdot \frac{1}{2}$$

½ ½ ½ ½

$$= 24 - 6 + 9\pi - \frac{9}{2}\pi$$

$$= 18 + \frac{9}{2}\pi$$ ½

[b] Evaluate $\int_{8}^{-10} f(x)\,dx$.

$$= -\int_{-10}^{8} f(x)\,dx$$

$$= -\left\{(24+9\pi) - \frac{1}{2}9\pi\right\}$$

$$= -\left(24 + \frac{9}{2}\pi\right) = -24 - \frac{9\pi}{2}$$

1 ½

A person's velocity (in meters per second) at time t (in seconds) is given by $v(t) = \begin{cases} 2t+7, & 0 \le t \le 3 \\ 16-t, & 3 \le t \le 15 \end{cases}$. m/s

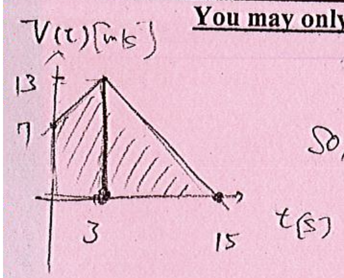
13 13

SCORE: ______ / 5 PTS

[a] Find the exact distance the person travelled from time $t=0$ seconds to $t=15$ seconds.

NOTE: You must show the arithmetic expression that you used to get your answer.

You may only use techniques discussed in sections 5.1 and 5.2.



The area shown left is the distance the person travelled.

So, Distance $= \int_0^3 (2t+7)\,dt + \int_3^{15} (16-t)\,dt = \frac{(7+13)3}{2} + \frac{1}{2}\cdot 12\cdot 13$ ½ ✱

$= 30 + 78$

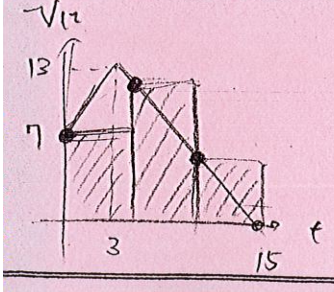
$= 108$

✱

A. 108 m ½

[b] Estimate the distance the person travelled from time $t=0$ seconds to $t=15$ seconds using three subintervals and left endpoints.

NOTE: You must show the arithmetic expression that you used to get your answer.



$\Delta t = \frac{15-0}{3} = 5$

Distance $= 5\{7 + (16-5) + (16-10)\}$ ½ 1½

$= 5(24)$

$= 120$

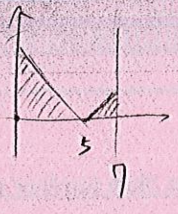
½ ½

A. 120 m

Evaluate $\int_0^7 (|x-5| - 4\sqrt{49-x^2})\,dx$ using the properties of definite integrals and interpreting in terms of area. **SCORE:** 5 / 5 PTS

NOTE: You must show the proper use of the properties of the definite integral, NOT just the arithmetic.

$$|x-5| = \begin{cases} x-5 & \text{if } x \ge 5 \\ -x+5 & \text{if } x \le 5 \end{cases}$$



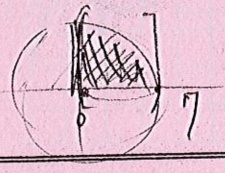
$$-\int_0^5 (x-5)\,dx = \frac{25}{2}$$

$$\int_5^7 (x-5)\,dx = 2$$

$$\int_0^7 (|x-5| - 4\sqrt{49-x^2})\,dx$$

$$= \int_0^7 |x-5|\,dx - 4\int_0^7 \sqrt{49-x^2}\,dx \quad 2$$

$$= -\int_0^5 (x-5)\,dx + \int_5^7 (x-5)\,dx - 4\int_0^7 \sqrt{49-x^2}\,dx$$



$$\int_0^7 \sqrt{49-x^2}\,dx = \frac{1}{4}\,49\pi = \frac{49\pi}{4}$$

Therefore, $\int_0^7 (|x-5| - 4\sqrt{49-x^2})\,dx$

$$= \frac{25}{2} + 2 - 4\cdot\frac{49\pi}{4} \quad \tfrac{1}{2} \quad \tfrac{1}{2} \quad 1\tfrac{1}{2}$$

$$= \frac{29}{2} - 49\pi \quad \tfrac{1}{2}$$

$4x(x+2)$ $\quad x(x+2)$ $\quad 4(x^2+2x)$ $\quad 4(x+1)^2$

Using the limit definition of the definite integral, and right endpoints, find $\int_{-1}^{5} (4x^2+8x)\,dx$. **SCORE:** ____ / 10 PTS

NOTE: Solutions using any other method will earn 0 points.

$$\int_{-1}^5 (4x^2+8x)\,dx$$

$$= \lim_{n\to\infty} \sum_{i=1}^n f(a+i\Delta x)\,\Delta x$$

$$\left(\Delta x = \frac{5-(-1)}{n} = \frac{6}{n} \right)$$

$$1 \quad \lim_{n\to\infty} \sum_{i=1}^n f\left(-1+\frac{6i}{n}\right)\frac{6}{n} \quad 1$$

$$= \lim_{n\to\infty} \sum_{i=1}^n \left[4\left(-1+\frac{6i}{n}\right)^2 + 8\left(-1+\frac{6i}{n}\right) \right]\frac{6}{n} \quad 1\tfrac{1}{2}$$

$$= \lim_{n\to\infty} \frac{24}{n} \sum_{i=1}^n \left[\frac{36i^2}{n^2} - \cancel{\frac{12i}{n}} + 1 + \cancel{\frac{12i}{n}} \cdot 2 \right]$$

$$= \lim_{n\to\infty} \frac{24}{n}\left[\frac{36}{n^2}\sum_{i=1}^n i^2 - \sum_{i=1}^n 1 \right] \quad *$$

$$= \lim_{n\to\infty} \frac{24}{n}\left\{ \frac{\cancel{36}^{\,6}}{n^2} \cdot \frac{1}{\cancel{6}}\,n(n+1)(2n+1) - n \right\} \quad * \quad 1$$

$$= \lim_{n\to\infty} \frac{24\cdot 6\,n(n+1)(2n+1) - 24n^3}{n^3}$$

$$= 24 \cdot 11$$

$$= 264 \quad 1$$